Journal of Engineering Physics and Thermophysics, Vol. 72, No. 5, 1999

DYNAMIC CHARACTERISTICS OF PULSATING TURBULENT FLOW OF COMPRESSIBLE GAS IN A CHANNEL

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A model of turbulent transfer that allows for the effect of flow-rate oscillations on the turbulent stress is used to investigate pulsating turbulent flow of a compressible gas in a narrow channel. An algorithm for solving numerically the system of equations that describes this process by the finite-difference method with the use of an implicit iteration scheme is proposed. The effect of operating parameters on the amplitude-frequency characteristic is considered.

Introduction. In construction of mathematical models of different pneumohydraulic systems that involve portions of pipelines with a moving compressible fluid as elements there arises the need for a detailed description of the processes of dynamics [1]. Thus, when the transfer functions of the channel, for example, the ratio of pressure (or flow-rate) oscillations at the inlet to the channel to these oscillations at the outlet are calculated, it is important to correctly allow for friction (the tangential stress) on the wall, which is determining both at low frequencies and especially at high frequencies when the amplitude of tangential-stress oscillations increases. The tangential stress on the wall for turbulent flow is found by solution of a system of the equations of fluid dynamics (the Reynolds and continuity equations) and the closing equations for Reynolds stresses.

In [2-4], consideration is given to the propagation of pressure waves in a channel for the case of the oscillations of a weakly compressible dropping liquid and adiabatic oscillations of a compressible gas. This process is described by a system of one-dimensional (averaged over the cross section of the channel) equations of motion and continuity with given boundary conditions for the pressure (the flow rate) at the inlet to and the outlet from the channel. It is shown in what cases pressure (flow-rate) oscillations can be represented as the sum of two longitudinal waves that propagate in opposite directions with an attenuation factor and phase velocity. The latter quantities are determined by the relative amplitude and the phase of the oscillation of the tangential stress on the wall, which is found by numerical (finite-difference) solution of the equation of motion within an approximation of a boundary layer or a narrow channel (it is shown, for which operating parameters of oscillating flow this approximation can be used). The turbulent stress involved in the equation of motion was found using a model of turbulent transfer that allows for the effect of unsteadiness. The validity of the model was confirmed by comparison of the results of calculating with the available experimental data for different types of unsteadiness both for an incompressible fluid and a compressible fluid [2-6]. The use of a quasistationary turbulence model leads to poorer worse agreement between the results of the calculation and experiment and in a number of cases makes it impossible to describe the effects observed in the experiments.

However, the assumptions that make it possible to directly relate the quantities characterizing the process of wave propagation to the friction on the wall are frequently not satisfied (for example, for large amplitudes of oscillations, when the equations cannot be solved in a linear approximation). To describe the indicated process, in this work we propose a model that involves the equation of motion, closing equations for turbulent transfer under unsteady-state conditions, and the equations of motion and continuity averaged over the cross section of the tube. We propose an algorithm for simultaneous numerical solution of this system by the finite-difference method using a stable implicit iteration scheme. Dynamic characteristics of the pipeline for adiabatic oscillations of the gas with large amplitudes are used as an example; the difference from the case of oscillations with small amplitudes is shown.

Moscow Power Institute (Technical University), Moscow, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 5, pp. 896-901, September-October, 1999. Original article submitted September 22, 1998.

1062-0125/99/7205-0865$22.00

The procedure of numerical modeling proposed can also be used when gas oscillations cannot be assumed to be adiabatic, for example, for problems of heat transfer in a pulsating flow of a compressible gas. In this case, it is necessary to additionally include the energy equation in the system of equations considered.

Formulation of the Problems. Let us write the equation of liquid motion in a circular tube in a boundary-layer approximation, disregarding convective terms (as shown in [2] this can be done for flows with Mach numbers $M << 1$):

$$\frac{\partial(\rho_{00} W_x)}{\partial t_\omega} = -\frac{\partial P}{\partial X} + \frac{1}{s^2}\frac{1}{R}\frac{\partial}{\partial R}\left[R\left(\frac{\partial W_x}{\partial R} + \tau_t \frac{\mathrm{Re}}{2u_0 w_0}\right)\right]. \tag{1}$$

To close Eq. (1), we must determine the turbulent tangential stress. For this purpose, we used a developed model of turbulent transfer based on relaxation equations for the turbulent stress and viscosity:

$$a_\tau t_\mathrm{T} \frac{\partial \tau_t}{\partial t} = \tau_\mathrm{eq} - \tau_t, \tag{2}$$

$$a_\varepsilon t_\mathrm{T} \frac{\partial \varepsilon_\tau^2}{\partial t} = \frac{\tau_t}{\tau_\mathrm{eq}} \varepsilon_\mathrm{eq}^2 - \varepsilon_\tau^2, \tag{3}$$

where $\varepsilon_\mathrm{eq} = l^2 \left|\frac{\partial w_x}{\partial r}\right|$; $\tau_\mathrm{eq} = \rho \varepsilon_\tau \frac{\partial w_x}{\partial r}$ are the equilibrium values of the corresponding quantities; $a_\varepsilon = 6.2$ and $a_\tau = 2.2$ are the relaxation constants. The procedure for calculating turbulent transfer under unsteady-state conditions are described in [5, 6] in detail.

Equation (1) is solved with the following boundary and initial conditions. When $R = 1$, $W_x = 0$; when $R = 0$, $\partial W_x / \partial R = 0$, and when $t_\omega = 0$, the initial velocity profile $W_x = W_x^0(R)$ is prescribed. Since consideration is given to a solution that is periodic over time, the form of the initial velocity profile has an effect only on the number of periods in which the solution is established (see below).

A solution of Eq. (1) is found in each cross section of the tube X from the known pressure gradient $\partial P/\partial X = f(X)$ and dimensionless density $\rho_{00} = f(X)$. We note that for adiabatic oscillations of the gas, whose parameters are described by the equation of state of an ideal gas (it is precisely this case that is considered in the present work), $\rho_{00} = (P/P_0)^{1/\gamma}$.

Thus, to solve Eq. (1), we must know the pressure distribution $P(X)$. It can be found by solution of the system of equations of motion and continuity averaged over the cross section of the tube:

$$\frac{\partial U}{\partial t_\omega} + \frac{\partial P}{\partial X} + E_\mathrm{c} = 0, \tag{4}$$

$$a_{00}^2 \frac{\partial U}{\partial X} + \frac{\partial P}{\partial t_\omega} = 0, \tag{5}$$

where $U = 2\int_0^l \rho w_x R dR / u_0$ is the dimensionless flow rate of the liquid (the average mass velocity); $F_\mathrm{w} = -\frac{2}{s^2}\left(\frac{\partial W}{\partial R}\right)_{R=1}$ is the friction on the wall, $a_{00}^2 = (P/P_0)^{1-1/\gamma}$.

To solve Eqs. (4) and (5), we must prescribe P and U at the inlet for $X = 0$ and the outlet for $X = X_\mathrm{ch}$ from the channel. The function F_w is found by solution of Eq. (1).

Therefore, the nonlinear equations (1), (4), and (5) should be solved simultaneously. The solution depends on the following operating parameters: the Stokes s and Reynolds Re numbers, the dimensionless pressure at the inlet P_0, the coefficient γ, the relative amplitude of pressure or flow-rate oscillations prescribed at either end of the tube, the form of the boundary conditions at the ends of the tube, and the tube length X_{ch}.

Method of Solution. For the approximation of Eqs. (4) and (5), use was made of an impicit scheme of the first order in t_ω that is symmetric in X and of the second order in X:

$$a_i^+ U_{i+1}^{n+1} - b_i U_i^{n+1} + a_i U_{i-1}^{n+1} = -f_i, \quad a_i^\pm = (a_{00}^2)_{i+1/2}^{n+1} (\Delta t/\Delta x)^2, \tag{6}$$

$$b_i = 1 + a_i^+ + a_i^- + (f_w)_i^{n+1} \Delta t, \quad f_i = U_i^n - D_i^n \Delta t;$$

$$P_{i+1/2}^{n+1} = P_{i+1/2}^n - (a_{00}^2)_{i+1/2}^{n+1} (U_{i+1}^{n+1} - U_i^{n+1}) (\Delta t/\Delta x); \tag{7}$$

$$D_i^{n+1} = (P_{i+1/2}^{n+1} - P_{i-1/2}^{n+1})/\Delta x. \tag{8}$$

Here $f_w = F_w/U$, $D = \partial P/\partial X$; $\Delta t = \Delta x$ are the t_ω and X steps; n is the number of the t_ω layer; i is the number of the X point.

System (6) for U_i^{n+1}, supplemented by finite difference equations that approximate the boundary conditions for $X = 0$ and $X = X_{ch}$ on each $n+1$ time layer, is solved by the running method. The running is stable and monotonic when $f_w > 0$.

Next, $P_{i+1/2}^{n+1}$ and D_i^{n+1} are found by (7) and (8). Since the coefficient $(a_{00}^2)_{i+1/2}^{n+1}$ in Eqs. (6) and (7) depends on $P_{i+1/2}^{n+1}$ it is approriate to use iterations. In the first iteration, $(a_{00}^2)_{i+1/2}^{n+1}$ is taken from the previous n layer. On the initial time layer, distributions $U(X)$ and $P(X)$ are prescribed. The number of periods in which a solution that is periodic in time is established depends on how similar these distributions are to the solution sought.

The function $(f_w)_i^{n+1}$, which is found by solution of Eq. (1), enters into the coefficient b_i of Eq. (6). To solve (1), we use a stable implicit scheme of the first order in t_ω and of the second order in R. On each $n+1$ time layer, the system of finite-difference equations for $(W_x)_j^{n+1}$ (j is the number of the point in R) is solved by the running method. The equations that determine the turbulent stress τ_t are integrated using an implicit scheme of the first order in t_ω. Since the equilibrium values of $(\varepsilon_{eq})_{j+1/2}^{n+1}$ involved in approximation (1) (indirectly, in terms of (2) and (3)) depend on the sought function $(W_x)_j^{n+1}$ we use iterations.

The coefficients ρ_{00} and $\partial P/\partial X$ in Eq. (1) on each time layer are determined by a solution of the system of one-dimensional equations of $P(X)$ and $U(X)$ obtained earlier. As the calculations showed, to satisfy the balance relations more accurately, it is appropriate to find the pressure gradient from the known value of $U(X)$ using the method of splitting.

We note that when the system of one-dimensional equations is solved the function f_w is taken from the previous n layer and then, on solution of Eq. (1), it should be refined. However this procedure would lead to a substantial increase in the volume of computations and was not performed in the present work. The fact that $(f_w)_i^n$ rather than $(f_w)_i^{n+1}$ is involved in Eqs. (6) has no effect on the stability and the order of approximation of the schemes, though it can lead to an increase in the error of solution. With the aim of decreasing this error we used linearization of the source in Eq. (4): $F_w = f_w U$. In many cases, for example, in the laminar regime of flow and in the turbulent regime in the region of high frequencies, f_w ceases to depend on time for oscillations with large amplitudes.

It should be noted that by employing a two-layer scheme that is symmetric in time for solution of the one-dimensional equations, we can bring the order of approximation up to the second order. In the present work, this is not performed, since Eq. (1) is solved using a scheme of the first order of approximation. When a two-layer symmetric scheme of the second order is applied to this equation in many cases the iterations do not converge, as calculations have shown.

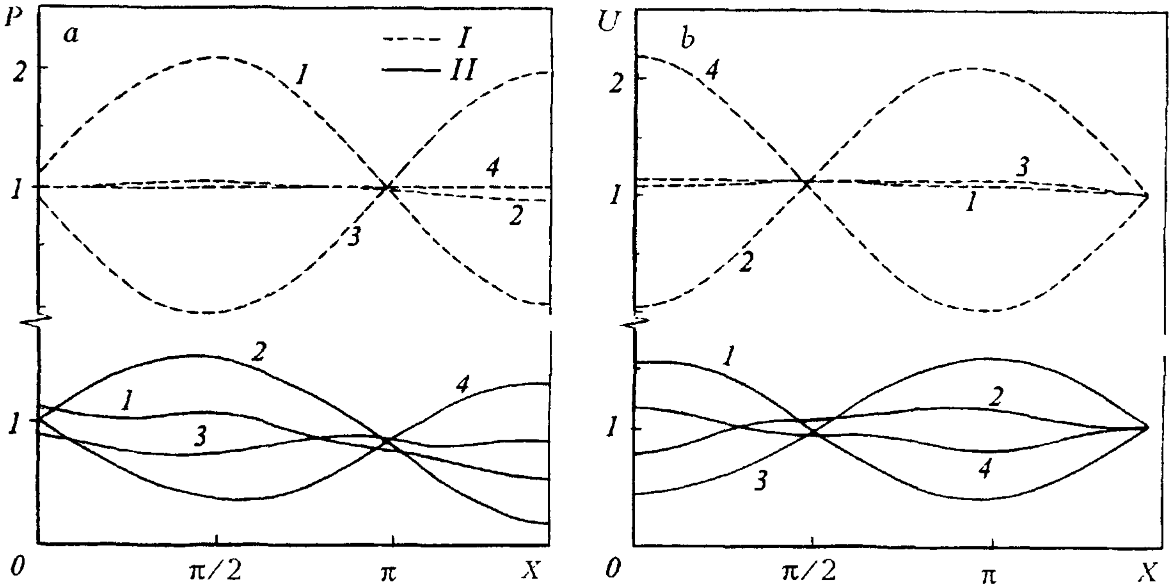


Fig. 1. Variation in pressure (a) and flow rate (b) along tube in different time phases: 1) $t_\omega = 0$; 2) $\pi/2$; 3) π; 4) $3\pi/2$ ($A_p = 0.1$, $X_{ch} = 2.9\pi/2$); *I*) solutions of acoustic equation; *II*) solution for gas, $\gamma = 1.4$, $P_0 = 1$, $Re = 10^5$; $L = 10^{-3}$.

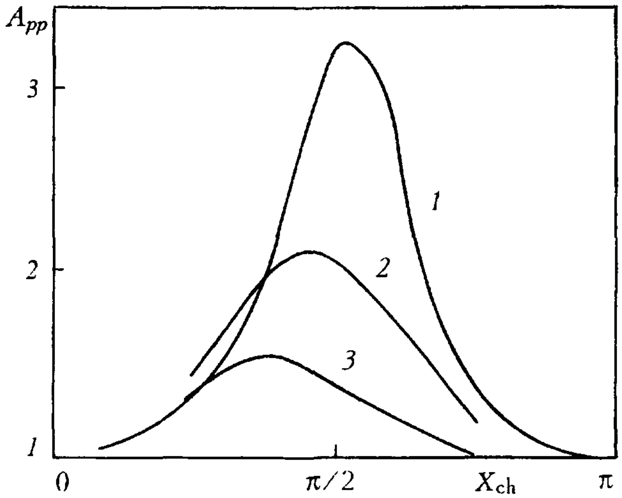
Results of Calculation. A solution is sought for the following boundary conditions at the inlet to and at the outlet from the channel: when $X = 0$ the pressure oscillations are prescribed in the form of the function $P = P_0(1 + A_p \cos t_\omega)$, where A_p is the relative amplitude of oscillations; when $X = X_{ch}$ the channel is closed: $U = 1$.

A feature of the process of wave propagation in channels is the presence of resonance regimes, in which, for certain dimensionless channel lengths X_{ch} (divisible by $\pi/2$ under these boundary conditions), we observe an increase in the amplitude of the oscillations of the pressure with $X \approx k\pi/2$ and of the flow rate with $X \approx (k - 1)\pi$; $k = 1, 2, \ldots$. In the limiting case of acoustic oscillations of a nonviscous fluid, the amplitude at the indicated points increases to infinity.

Figure 1 shows the calculated distributions of the pressure and the flow rate along the tube length for one of the regimes in the near-resonance region. As compared to the results of solving the acoustic equations, the shape of the curves is distorted substantially, the amplitude of oscillations decreases, a pronounced decrease in the pressure along the length is observed, and an X-dependent shift of the phase of oscillations relative to the phase of the pressure at the inlet appears.

As can be seen from the figures, when the above finite-difference scheme is applied to solution of the acoustic equations, there is an error due to the scheme viscosity. The error can be reduced by decreasing the step Δt or by switching to the scheme of the second order of accuracy in t_ω. However, in calculating the oscillations of a viscous fluid the role of the scheme viscosity becomes insignificant. Thus, the results of the calculation of transfer functions for the limiting case of small amplitudes of oscillations according to the above scheme coincide (with an error of several percent) with the results obtained in [14], where use was made of the close relationship between the transfer functions and the relative amplitude and the phase of oscillations of the tangential stress on the wall. From the results indicated above, we determined the optimum parameters of the scheme: the number of splittings per oscillation period and the length of a longitudinal wave was 720 and along the radius (with logarithmic bunching near the wall) it was 100. It took 3 to 16 periods (depending on operating parameters) to establish a flow that was periodic in time.

In [4], the amplitude frequency characteristics – the ratio of the amplitudes of pressure oscillations at the outlet and the inlet A_{pp} as a function of X_{ch} – were calculated for the oscillations of a weakly compressible dropping liquid with small amplitudes. The results obtained are in agreement with the results of experiment of [1]. The indicated dependences have resonance character with pronounced maxima at $X_{ch} = k\pi/2$. The magnitude of the



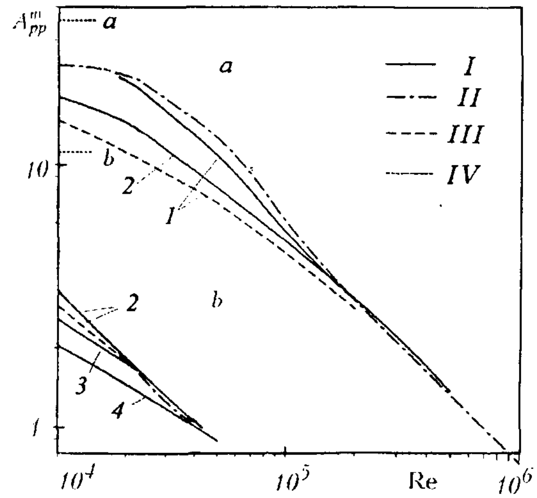


Fig. 2. Amplitude frequency characteristic in the vicinity of the first resonance harmonic ($L = 10^{-2}$: 1) for a liquid with small amplitudes of oscillations, Re = 10^4; 2 and 3) for a gas, $\gamma = 1.4$, $P_0 = 10$, and $A_p = 0.2$, 2) Re = 10^4; 3) $2.25 \cdot 10^4$.

Fig. 3. Effect of operating parameters on the resonance amplitude of oscillations of the pressure at the outlet (a) $L = 10^{-3}$; b) 10^{-2}): *I* and *III*) for a gas ($\gamma = 1.4$: 1) $A_p = 0.04$ and $P_0 = 1$; 2) 0.04 and 5; 3) 0.2 and 5; 4) 0.2 and 10); *II*) for a liquid ($A_p = 0.01$); *III*) results of calculation by quasistationary turbulence model; *IV*) limiting values for high-frequency region.

maximum depends on the number of the resonance harmonic k and on the Reynolds number and the reduced length of the tube $L = x_{ch}\mu/\rho_0 a_0 r_0^2$), $L = X_{ch}/s^2$. The same characters of the dependences is also retained for adiabatic oscillations of the gas (see Fig. 2), however, in this case, the resonance amplitude of the oscillations A^m_{pp} is noticeably lower. Furthermore, a decrease in the resonance value of X_{ch} as compared to $\pi/2$ is observed. Indeed, with large amplitudes the oscillations are nonlinear and higher harmonics both in time and in length occur. In this case, we used the average amplitude calculated from the arithmetic mean of the maximum and minimum deviations from the period-averaged pressure at the outlet as A_{pp}. It should be noted that we were unable to compare the results obtained with experimental data for the gas, since such data are unknown to the author.

The effect of different operating parameters on A^m_{pp} for the first resonance harmonic is shown in Fig. 3. An increase in the amplitude of oscillations A_p and the average pressure P_0 at the inlet leads to a decrease in A_{pp}. For small Re and L, the results of the calculation depend substantially on the turbulence model used. In the high-frequency region (large Stokes numbers s), a transition to the regime of frozen turbulence is realized [4, 6], in which the characteristics of the oscillating flow obey the regularities of laminar flow. The figure shows the limiting values of A^m_{pp} for this high-frequency regime, which depend only on L (and on the number of the resonance harmonic) with small amplitudes of oscillations. In using the quasistationary turbulence model (without relaxation equations for the turbulent stress and viscosity $\tau_t = \tau_{eq}$ and $\varepsilon_\tau = \varepsilon_{eq}$), the transition to the regime of frozen turbulence is delayed. The values of A^m_{pp} turn out to be similar to the values for the region of quasistationary turbulence rather than to the high-frequency limit [4].

Conclusion. The possibility of applying the finite difference method to solution of a system of equations that describes the turbulent pulsating flow of a compressible gas in a narrow confined channel when the process of propagation of pressure and flow-rate waves is effected substantially by the friction on the wall is shown. A solution is obtained for cylindrical geometry, but the method is easily extended to the case of flow in a plane channel; the basic results will not change fundamentally.

The calculations of the amplitude frequency characteristic performed in the near-resonance region confirmed that the nonlinearity of oscillations leads to a decrease in the amplitudes of the oscillations along the entire length of the channel.

The use of relaxation equations that allow for the effect of flow-rate pulsations on the turbulent stress is fundamental in describing turbulent transfer. When the quasistationary turbulence model (without relaxation equations) is used for small Reynolds numbers in the region of high frequencies (large Stokes numbers), the amplitudes of oscillations turn out to be understated.

NOTATION

t, time; ω, angular frequency of oscillations; x and r, axial and radial coordinates; ω_x, axial velocity; p, pressure; ρ, density; μ, dynamic viscosity; a, velocity of sound; γ, adiabatic exponent; r_0, radius of tube; u_0, time-average flow rate of liquid (average mass velocity); ρ_0 and a_0, density and velocity of sound averaged over the period of oscillations in the initial cross section of the tube ($x = 0$); $t_\omega = t\omega$; $R = r/r_0$; $X = x\omega/a_0$; $W_x = w_x/w_0$; $w_0 = u_0/\rho_0$; $P = p/(u_0 a_0)$; $\rho_{00} = \rho/\rho_0$; $a_{00} = a/a_0$; P_0, dimensionless period-average pressure in the initial cross section of the tube; $s = r_0\sqrt{\omega\rho_0/\mu}$, Stokes number; $\mathrm{Re} = 2u_0 r_0/\mu$, Reynolds number; τ_t and ε_τ, turbulent stress and viscosity; t_T, time scale of turbulence; l, geometric scale of turbulence (mixing length).

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